EXAMINING PRESERVICE TEACHER THINKING ABOUT TECHNOLOGY-BASED TRIGONOMETRIC EXPLORATIONS THROUGH A REPLACING, AMPLIFYING, AND TRANSFORMING FRAMEWORK

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Researchers promoting the inclusion of technology for teaching and learning have recently called for the integration of mathematical technologies into the preparation of future teachers. This report analyzes the dynamic geometry sketches produced by preservice secondary mathematics teachers when investigating trigonometric relationships. We analyzed preservice teachers' approaches to a particular task based on the extent to which the technology replaced, amplified, or transformed learning opportunities about tangent. We highlight the prominent aspects of the diagrams produced and discuss technological affordances these diagrams present for supporting preservice teachers' development of content knowledge and pedagogical approaches related to trigonometry.

Keywords: Technology, Teacher Education – Preservice, Geometry and Geometrical and Spatial Thinking

There have been recent pushes to incorporate technology into the teaching and learning of K-12 mathematics (National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). As Lee and Hollenbrands (2008) point out, "Whether technology will enhance or hinder students' learning depends on teachers' decisions when using technology tools, decisions that are often based on knowledge gained during a teacher preparation program" (p. 326). Subsequently pre-service teachers (PSTs) need opportunities to incorporate technology to learn and teach mathematics (Association of Mathematics Teacher Educators, 2006; International Society for Technology in Education, 2008). Incorporating technological tools into mathematics content courses allows conversations about new ways in which mathematical topics can be explored and new mathematical investigations that were not previously possible with more traditional tools (Hughes, Thomas, & Scharber, 2006).

One mathematical topic PSTs particularly struggle with is trigonometry (Moore, 2009), which spans both geometric and algebraic perspectives where an algebraic approach uses the unit circle as the object of study and a geometric approach uses right triangles as the object of study. Although researchers advocate for students to make connections between these two approaches, teachers often have difficulty coherently understanding trigonometric relationships (Moore, Paoletti, & Musgrave, 2014; Weber, 2005). This paper examines a set of activities used to develop 20 secondary preservice mathematics teachers' thinking about trigonometric relationships. These tasks required teachers to create dynamic geometry sketches in Geogebra or Geometer's Sketchpad to create multiple representations of trigonometric relationships and make connections between these representations. Specifically, the activity asked the PSTs to examine the slopes of lines, the slope-triangles created on these lines, the ratios of side lengths within these triangles, and the angles within these slope triangles. This research examines the following question: How does the production of dynamic geometry sketches by preservice teachers support their understandings of tangent, and what is the role of technology (i.e. *replace*, *amplify*, *transform*) in supporting their content knowledge about the tangent relationship?

Frameworks

The Technological Pedagogical Content Knowledge (TPACK) framework, described by Koehler and Mishra (2009) unpacks the unique kinds of knowledge that go into the effective teaching of

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content (in this case mathematics) with technology. The framework identifies ways in which pedagogical, technological, and content knowledge interact with each other in a given context. The emergent forms of knowledge are "the basis of effective teaching with technology, requiring an understanding of the representation of concepts using technologies" (p. 66). This framework is also a useful lens for examining the technological artifacts that teachers create and use to explore mathematical content. Through these artifacts, mathematical, pedagogical, and technological issues are highlighted and minimized when they are used as tool to illustrate and investigate mathematics. By investigating the technological tools created by PSTs, we gain insight into their understanding of the complex interactions between technology, pedagogy, and content.

As new technologies are developed and utilized in mathematics classrooms their effects commonly *replace*, *amplify*, or *transform* non-technological means of teaching the same content (Hughes et al., 2006). New technological features (e.g. dynamic presentations, simultaneous representations, communication, etc.) offer the potential to change the mathematical explorations and discussions that occur in classrooms. However, this is only possible if the technology is used in ways that make use of this potential. The Replacing, Amplifying, and Transforming (RAT) framework provides a lens for analysis of the mode of technology integration into an activity (Hughes et al., 2006). This framework draws attention to whether the technology *replaces* a similar presentation without the technology, *amplifies* the learning process that was present in the non-technology version, or *transforms* the learning experiences to provide possibilities that were otherwise not possible without the technology. This framework has been a useful tool for analyzing the integration of technology into teacher preparation courses as a means for understanding how teacher educators can improve instruction (Glassmeyer, Brakoniecki, & Amador, 2016a, 2016b). The RAT framework allows for the analysis of technology and the range of ways it is used in tasks to advance learning outcomes.

The tasks used with these PSTs were focused on exploring mathematical content, namely the tangent relationship (as these tasks were given in a mathematics content course for beginning teachers). In our study, the TPACK framework is a theoretical lens for understanding how PSTs conceptualized the tangent relationship while using technology tools and considering their future careers as secondary mathematics teachers. The RAT framework functions as an analytic lens to describe how the PSTs used sketches when exploring this relationship and how the technology provided for learning opportunities. Together, these frameworks for technology integration provided understanding about how PST-generated dynamic geometry sketches supported their understandings of tangent as technology was integrated into a given task.

Method

This study took place at a large Southern university within a content course for prospective secondary teachers. In the course, PSTs regularly engaged in learner-centered instruction incorporating collaborative learning and technology such as graphing calculators, Geometer's Sketch Pad, Geogebra, and Desmos. Approximately half of the course meetings were devoted to having PSTs explore trigonometric relationships to support their quantitative reasoning and ultimately their conceptual understanding of mathematical topics, with secondary attention going to enhancing their pedagogical content knowledge by considering how they would teach this material to their own students.

The study focuses on a three-day lesson where the 20 PSTs investigated the tangent relationship. The task was modified from the CPM Core Connections Geometry textbook (Kysh, Dietiker, Sallee, Hamada, & Hoey, 2013) by requiring teachers to create dynamic geometry sketches to answer parts of the lesson. The lesson has teachers explore connections between the slope-ratio triangle (a right triangle produced by a line and it's vertical-to-horizontal change) and the base angle of that slope triangle angle. (See figure 1 for an example of a slope ratio triangle in a pre-constructed applet).

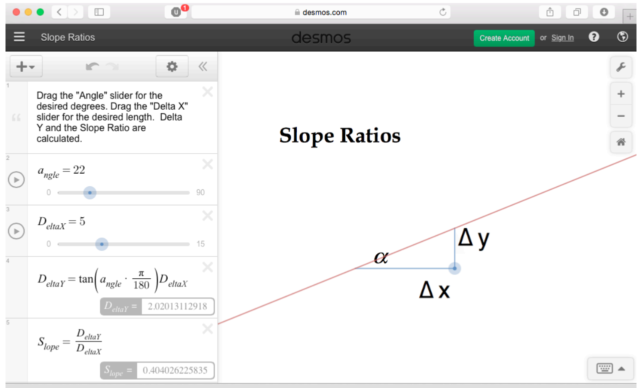

Figure 1. Screenshot of an app that can be used to explore relationships of slope and angle.

Near the beginning of this exploration, the PSTs determined (with a paper and pencil task) that for a slope-ratio triangle with an 11º base angle, the slope ratio is approximately 1/5. Additionally, for a triangle with a 22º base angle, the slope ratio is approximately 2/5. These beginning teachers were asked to construct their own dynamic geometry sketches to explore this supposed fact from their pencil and paper sketches.

Data for this study include PST homework and subsequent class discussions in which they used the dynamic geometry software to explore the tangent relationship. The preservice teachers were explicitly asked to create dynamic geometry sketches where they could "click and drag a point to get different slope triangles." The PST then met in class and presented their sketches to their peers. These whole class discussions were audio recorded as well as conversation of three small groups of 3-4 PSTs as they worked through the task. The series of tasks lasted three days, resulting in a total of nine transcripts, three small group recordings a day for three days. As a part of a larger project, the transcripts were initially read by all three researchers of the project, one being the instructor of record for the course in which the data were collected. The data were originally open coded using Strauss and Corbin (Corbin & Strauss, 2007) constant comparative methods by each of the researchers. The researchers then met and identified technology as an emergent theme in the data as a way to support the PSTs' understanding. Following this, cognizant of the theoretical framing of TPACK (Koehler & Mishra, 2009) and the analytic framework for RAT (Hughes et al., 2006) the three researchers each independently recoded the entire data set with a focus on the role of technology in supporting PSTs' understandings of the tangent relationship. The identified chunks in the data that related to technology and assigned a code of *replace*, *amplify*, or *transform*. The three researchers then met and compared codes of the data. Themes related to technology use were then derived and agreed upon. The researchers then collectively generated themes around the *replace*, *amplify*, and *transform* uses of the technology as related to learning tangent to describe the PSTs' use of the dynamic geometry software for exploring the tangent relationship. An emphasis was on understanding variations of technology use to support understanding. Finally, the dynamic geometry sketches were analyzed to corroborate findings to further understand how PSTs were using technology to understand the tangent relationship.

Findings

This section describes several of the variations among the multiple sketches produced by the PSTs. Each variation is discussed including its mathematical, pedagogical and technological implications, with specific emphasis on the role of technology to *replace*, *amplify*, or *transform* learning processes.

Static vs. Dynamic

One of the features of dynamic geometric software is the ability to drag objects in a sketch and observe what happens to the relationships among lengths, angles, and the connected shapes. This provides an *amplification* over static sketches where multiple sketches must usually be created in order to observe a change in various relationships. In this study, some of the sketches produced by the PSTs contained no dynamically moving parts. Either this was because all pieces of the sketch were "locked" together (any attempt to move a single aspect of the sketch actually moved the entire sketch), or there was no dynamic and relational aspects included in a picture (e.g. lengths of sides were written in, not linked to actual lengths, there were no preserved aspects of the diagram evident such as lines remaining horizontal or vertical)—in other words, these uses *replaced* traditional methods for learning. Here, PSTs treated their sketch the same as a regular pencil and paper sketch, only done on a computer, choosing not to utilize one of the key advantages of dynamic geometry software. This choice may have come from a lack of technological knowledge around how to display these measures or construct these relationships within their sketches. Additionally, their understanding of the activity may have centered on producing a sketch that displayed the relationship as opposed to exploring the relationship.

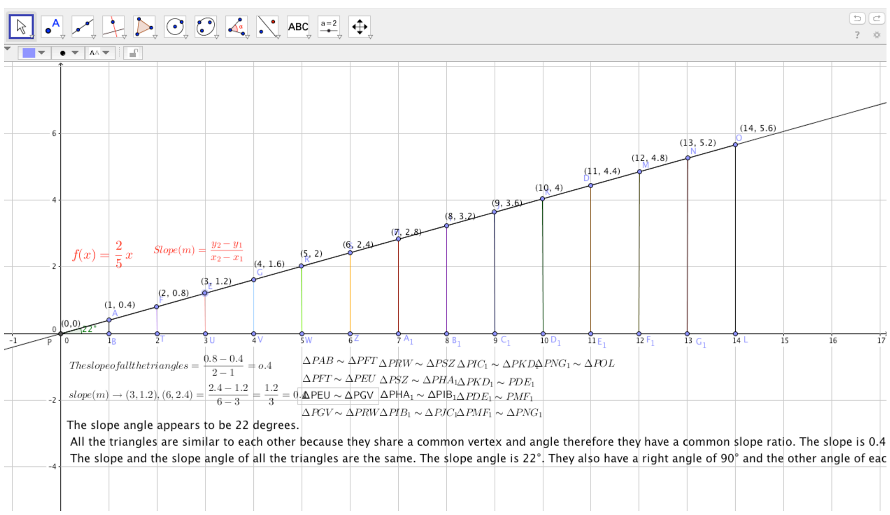

Figure 2. Screenshot of PST-created sketch with multiple slope triangles.

Multiple Overlapping Triangles

When using dynamic geometry software, the dragging of points and lines often allows the user to, in essence, see multiple different arrangements of figures within quick succession of each other. While at any given time, only one version of the figure is visible, the moving of one aspect allows users to visualize patterns and relationships in the figure and explore what may vary or remain invariant. With paper and pencil static sketches, often multiple versions of the figure will be included in a single diagram, with appropriate aspects labeled, in an attempt to "illustrate" these relationships. Some of the PSTs in the study created dynamic sketches and also included multiple slope triangles in their diagrams (see figure 2). While it was possible to dynamically manipulate their sketches to see relationships, the existence of multiple triangles made this interaction unnecessary as the relationships for consideration were already displayed upon opening the sketch. Here, again, the sketches seemed to be digital representations of static diagrams *replacing* a sketch of a similar representation, and the pedagogical advantage of exploration with dynamic geometry software was no longer a key feature of these sketches.

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Right Angle

When creating dynamic sketches, the objects are often created in a parent-child relationship. For example, when creating a line segment, two points are first placed (the parents) then the line segment (the child) connects the two points. Whenever a parent is moved, objects that are dependent on the parent are adjusted accordingly. With many digital sketches, there are often aspects of a diagram that users would like to vary, and other aspects of a diagram that creators may wish to remain invariant. For example, when investigating parallelograms, a learner may wish to have the side lengths and angles be modifiable, but always have the opposite sides of this quadrilateral be parallel and congruent in length. For the activity in our task, one of the crucial elements of a slope triangle is comparing the “rise” to the “run” of the triangle, which must be at right angles to each other. In many of the PST-generated sketches, the sketch of the slope triangle initially showed a right triangle. However, when vertices of the triangle were moved with the technology, their triangles no longer remained right triangles (see figure 3 for a sketch before and after manipulation). When these PSTs created their triangles in the software, the right angle was not an invariant part of the diagram. When the right angle was set to be invariant, the sketch *amplified* what might have been presented in other static representations, making it clearer for those looking at the sketch the relationships that exist in the diagram. It is possible that these PSTs did not know how to arrange the parent-child relationships among the aspects of their, diagram which would ensure that the angle of the slope-ratio triangle always remained at 90º. Additionally, they might not have understood the importance of this feature of their diagram. Consequentially, they may have felt that by closely approximating a right angle in their diagram through the manipulations of multiple points, you could still see the relationship between side lengths, and the base angle of the slope-ratio triangle. However, this introduced a potential source of error in measurements and the conclusions based off of those measurements, in addition to being a less efficient way of moving vertices and preserving the right angle.

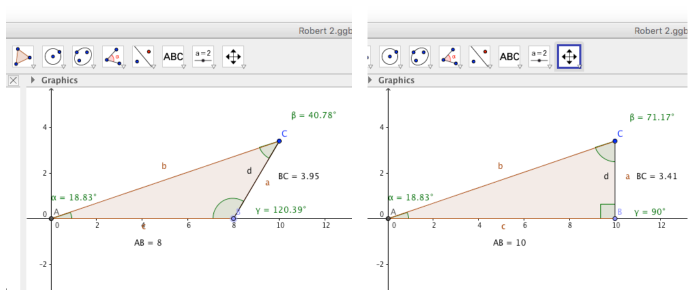

Figure 3. Screenshot of PST-created sketch with a non-preserved right angle.

Fixed Angle vs. Fixed Slope

In this activity, PSTs were asked to explore the relationship between the degree of the base angle in the slope-ratio triangle and the slope-ratio of that triangle. To create these slope-ratio triangles, there were two approaches PSTs used. One approach was to use a horizontal base for the triangle, and construct a line off of that base at a given angle for the hypotenuse of the triangle. In these sketches, the exploration allowed PSTs to see, given a particular degree measure, approximately what slope-ratio results exists in that triangle? A second approach in constructing these triangles was to again, begin with a horizontal base length. To create the hypotenuse, a line defined by a function with a given slope was created.

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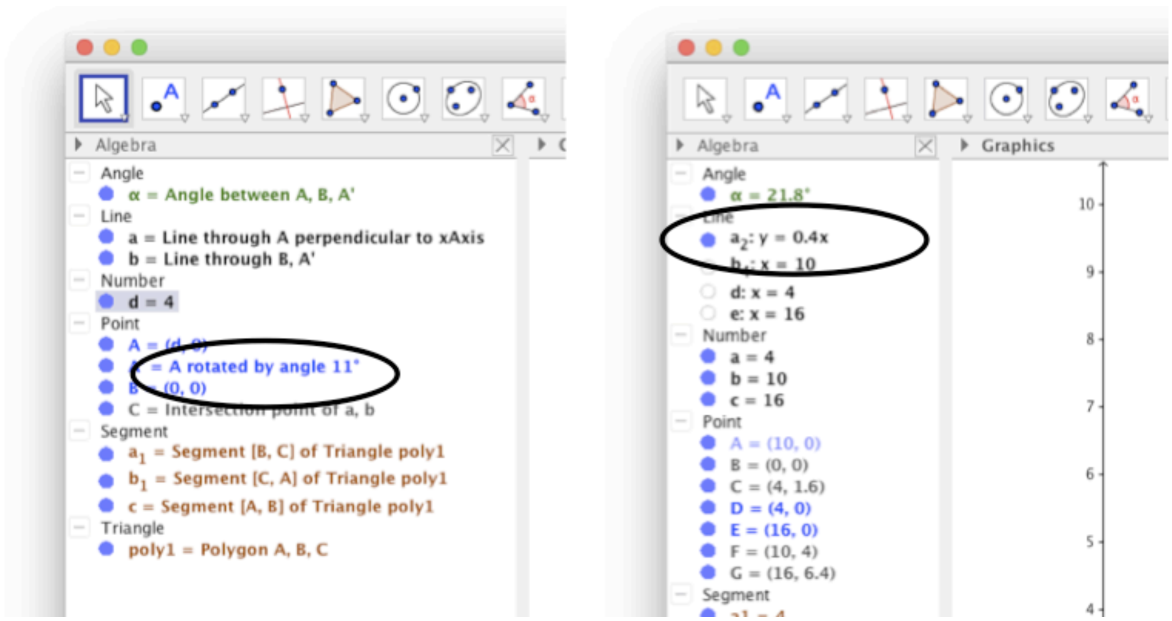

Figure 4. Screenshot of PST-created sketches with triangle created by angle, and by linear function with slope.

With these diagrams, the exploration allowed PSTs to see, given a triangle with a particular slope ratio, what is the base angle of that slope ratio triangle? Figure 4 provides an example of each approach used by the same PST in different sketches. The circled aspect of each diagram highlights how they defined their slope-ratio triangle. One of the key features to note with this different approach to constructing the triangles is that it actually enabled different mathematical explorations, in essence *transforming* the learning process in mathematics by allowing users to retrace exactly how a sketch was created. Furthermore, these different approaches emerged from the same activity around exploring the relationship. It's unclear whether the PSTs were aware of how their constructions impact the mathematics that they were investigating, or how these two approaches are complementary and can work together to support the investigation of the same relationship.

Unit Circle

This overall activity focuses on investigating right triangle trigonometry (relationships between the angles and side lengths in a right triangle). In addition to this perspective, trigonometry can also be investigated with the unit circle (a circle of radius 1 unit). In this approach, often times triangles are drawn inside of these unit circles, with the hypotenuse of each triangle extending from the origin to the circle, and the triangle is drawn connecting the point on the hypotenuse/unit circle to the x-axis so it intersects perpendicularly. In these unit circle triangles, the vertical length of this triangle is the sine of the angle, and the horizontal length of this triangle is the cosine of the angle. In some of the sketches produced by the PSTs, there appeared to be some efforts made to merge these perspectives of trigonometry. Figure 5 provides one instance of this approach by a PST. In these sketches, the slope-ratio triangle was embedded in circles centered at the origin (although not always unit circles). This attempt at merging was important for several reasons. First, it was an attempt to bridge mathematics content, usually presented differently in two domains (algebra and geometry), in essence *transforming* the learning process in mathematics. While there are mathematical similarities in these perspectives, the differences shape the range of relationship explorations.

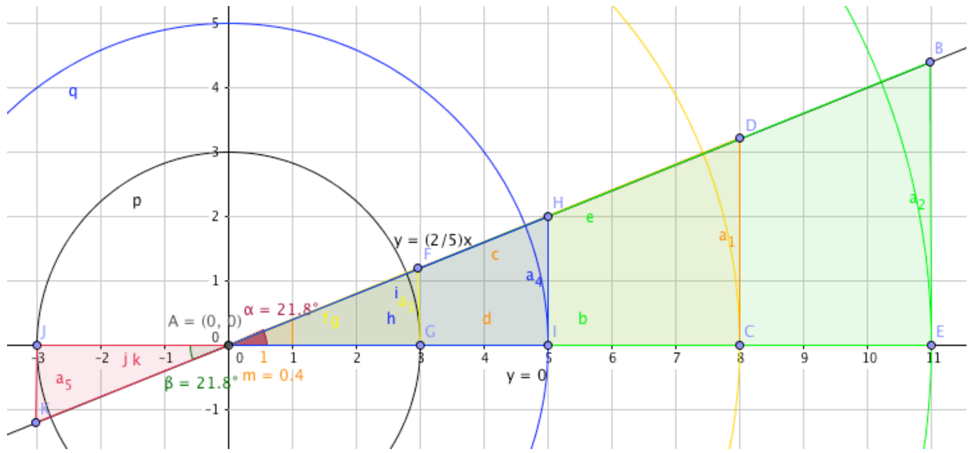

Figure 5. Screenshot of PST-created sketches with slope triangles and circles.

Right triangle trigonometry is limited to exploring angles between 0º and 90º while unit circle trigonometry is presented only with triangles with a hypotenuse of 1 unit. Additionally, the sketches that merge the right triangle and unit circle approaches have the potential for helping PSTs understand the strengths and limitations of each perspective and also offer ways that PSTs can simultaneously draw upon both perspectives when reasoning through a problem.

Discussion

Through this task of exploring the slope ratio triangle and its angles using dynamic geometry software, the PSTs were exposed to different approaches for thinking about trigonometric content and the tangent function and had opportunities to use technology in ways that would *replace*, *amplify*, or *transform* their previous experiences (Hughes et al., 2006). The activity was designed to engage these beginning teachers in mathematical explorations, but to also encourage them to think about the use of this task with their own students, and use technology to explore and explain some of the conjectures they were making. In analyzing the diagrams produced by the PSTs, we uncovered the variety of approaches used in their constructions, the role of the diagram with the activity, and the mathematical, pedagogical, and technological choices made by these PSTs. Specifically, they were able to recognize the role of technology in affording opportunities to *transform* their previous experiences through technological manipulation that otherwise would not have occurred with a pencil and paper drawing.

This study raises points about our work with, and study around, the role of technology and dynamic geometry programs with PSTs. As teacher educators, we need to be more explicit with PSTs about the strategic advantages that individual technologies offer (e.g. dynamic movement, simultaneous changes in representation, etc.) and how they may transform learning for their students in ways analogous to the transformations that occurred within the context of this secondary content course. One aim is to ensure that PSTs doing work to incorporate technology in their teaching are aware of the ways that the technology is best suited to enhance their instruction, albeit through replacement, amplification, or transformation of learning processes (Hughes et al., 2006). This may involve explicitly focusing different forms of TPACK knowledge (Koehler & Mishra, 2005) PSTs might be drawing upon when they create and use these sketches or other technologies. As we look toward supporting future teachers' uses of technology for their own teaching and learning, we recognize the complex interactions of mathematics content, the incorporation of technology into practice, and the attention to the ways in which learners will interact with that technology. By understanding how PSTs are currently making sense of these interactions, we hope to better support their future efforts.

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